Session 3
Extensions of Quantile-type Regression: Handling Hierarchical Structures

Nikos Tzavidis (University of Southampton)
Ray Chambers (University of Wollongong)
James Dawber(University of Wollongong)
Nicola Salvati (University of Pisa)

Pisa, July, 2016

Introduction

- In previous sessions we learnt about quantile, M-quantile and expectile regression
- However, until now we have assumed that the observations are independent
- A more realistic scenario is to assume that sample data has been collected via a complex survey design
- In this session we will deal with two types of data structures
  - Multilevel data
  - Longitudinal data

Outline

- The importance of data structures
- Quantile Multilevel Regression: Motivating research questions
- Quantile random effects regression by using the ALD
- M-quantile and expectile random effects regression
- A simulation study
- A case study: Longitudinal analysis of child psychopathology outcomes in the UK
- *R* Software

The Importance of Data Structures

- Until now we have assumed that data is independent
- Data in the real world has structure that tends to violate the assumption of independence
- Structures are generated by
    - Data collection mechanism
    - Natural structures within the population
    - Longitudinal data is a combination of both

The Independence Assumption

- Survey data rarely comes from a Simple Random Sample
- True for social surveys as these often involve multi-stage designs
- Cost advantages and it is often necessary when there is no suitable frame to sample households (or individuals) directly
- Outcome: Clustered data
- An even bigger issue with longitudinal data as the clustering occurs at the unit level

Examples

- Pupils within classes within schools
- A pupil's performance not only depends on their characteristics but also on the class and the school characteristics
- Individuals within households within communities
- Patients within wards within hospitals
- Weight measures within individuals within families (Repeated Measures)

Notation

- Outcome for individual i in group k, y_{ik}
- Refer to the i as level one and k as level two
- Can be extended to include further levels

Modelling Approaches

- Traditional approaches to analysing hierarchical data treat clustering as a nuisance that must be accounted for
- Parameters are estimated in the usual way but standard errors are adjusted for the impact of the clustering
- Model-based approach: build a model that represents the population from which the data was selected
- Impact of clustering is no longer just a nuisance but is of substantive interest in its own right

Modelling Hierarchical Data

- Possible Choices:
  - Standard models with robust standard errors
  - Aggregate Analysis
  - Disaggregate Analysis
  - Ignore the problem
  - Fixed effects
  - Multilevel models with random effects

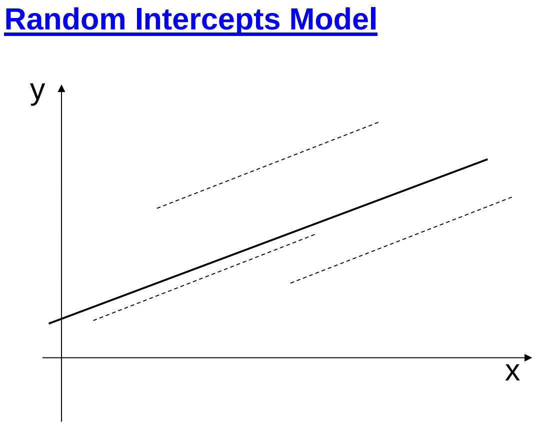
Random Effects Model

- Simplest model - Random Intercepts

$$y_{ik} = x_{ik}^T\beta + u_k + \epsilon_{ik}$$

- u_k is random, e.g. $u_k \sim N(0, \sigma_u)$
- $\epsilon_{ik} \sim N(0, \sigma_\epsilon)$
- σ_u quantifies group differences
- u_k is a group effect after controlling for x
- $u_k = 0$ average group
- $u_k > 0 (u_k < 0)$ above (below) average group

Graphical Representation - Random Intercepts



Quantile Multilevel Regression: Motivating Research Questions

- Pain management: How does the level of pain change over time in patients who received a treatment? (Geraci & Bottai, 2007)
- Does it change at the same rate for those who are more (less) resilient to pain?
- Does prior exam performance affect current exam performance similarly for the best and least well performing students?
- What is the impact of literacy on economically deprived households in a given geographical area?

Quantile Multilevel Regression

Details

Asymmetric Laplace: A continuous r.v $y \sim \; ALD(\mu, \sigma, q)$

$$f(y|\mu,\sigma,q) = \frac{q(1-q)}{\sigma} exp\Bigg(-\rho_q\Bigg(\frac{y-\mu}{\sigma}\Bigg)\Bigg)$$

- Quantile Nested Error regression (QNER) model (Geraci & Bottai, 2007,2014)
- $f(y,u|\beta,\sigma,\Gamma) = f(y|\beta,\sigma,u)f(u|\Gamma)$
- $y|u \sim ALD(x^T\beta_q + u, \sigma, q)$
- $p(u|\Gamma)$, Normal (Geraci & Bottai, 2007,2014)
- $p(u|\Gamma)$ Non-parametric via discrete mixture (Marino, Alfo & Tzavidis, 2015)
- Estimation of β_q, σ, Γ via MLE

Quantile Multilevel Regression - Practical Issues

- Software is available in R via library lqmm (Geraci and Bottai, 2013)
- Allows for random slopes and random intercepts, hence both hierarchical and repeated measures data can be modelled
- Does not handle 3-level structures
- Inference is via bootstrap and it can be time consuming

M-Quantile Multilevel Regression

Details

- Based on an extension of ML II approach of Richardson & Welsh (1995, Biometrics), Sinha & Rao (2009, CJS)
- Estimating equations for fixed effects and the variance components

$$x^T V_q^{-1} U_q^{1/2} \psi_q\{r_q\} = 0$$

$$\frac{1}{2}\psi_q\{r_q\}^T U_q^{1/2} V_q^{-1} Z Z^T V_q^{-1} U_q^{1/2} \psi_q\{r_q\} - \frac{K_{2q}}{2} tr\left[V_q^{-1} Z Z^T\right] = 0$$

$$\frac{1}{2}\psi_q\{r_q\}^T U_q^{1/2} V_q^{-1} V_q^{-1} U_q^{1/2} \psi_q\{r_q\} - \frac{K_{2q}}{2} tr\left[V_q^{-1}\right] = 0$$

M-Quantile Multilevel Regression

Links with Alternative Methods

- Estimating equations of the multilevel model obtained as a special case for $q = 0.5$ and a squared loss function
- For $q = 0.5$ and a loss function such as ρ_{Huber} recovers Robust ML II
- Expectile regression via the use of a large tuning constant

M-Quantile Multilevel Regression - Practical Issues

- Estimation for fixed effects: Newton Raphson
- Estimation for variance components: Fixed point algorithm
- Software is available in R (Tzavidis et al., 2016)
- Allows for random intercepts only
- Has been extended to handle 3-level structure (Borgoni et al., 2015)
- Prediction of random effects via extension of Fellner (1986, Technometrics) estimating equation
- Analytic expressions, via Taylor expansion, for computation of standard errors

Simulation Study

- Aim: Compare the M-quantile random effects approach to alternative approaches
- Simulation model

$$y_{ik} = 100 + 2x_{ik} + u_k + \epsilon_{ik}, i = 1, \ldots, n_k, k = 1, \ldots, 100,$$

- Scenarios
  - $[N, N]$ - Normal distributions: $u \sim N(0, 3)$ & $\varepsilon \sim N(0, 5)$
  - $[T, T]$ - t-distributions: $u \sim t(3)$ & $\varepsilon \sim t(3)$
  - $[N, Lap]$ - Normal and Laplace distributions: $u \sim N(0, 3)$ & $\varepsilon \sim Laplace(0, scale = 1.58)$
  - $[u, \varepsilon]$ - Outliers in both hierarchical levels generated via a contamination mechanism: $u \sim N(0, 3)$ for $k = 1, \ldots, 90$, and $u \sim N(0, 20)$ for $k = 91, \ldots, 100$, $\varepsilon \sim 0.9N(0, 5) + 0.1N(0, 150)$

Performance Measures

(a) Average Relative Bias (ARB) defined as

$$ARB(\hat{\theta}) = R^{-1} \sum_{r=1}^{R} \frac{\hat{\theta}^{(r)} - \theta}{\theta} \times 100,$$

$\hat{\theta}^{(r)}$ is the estimated parameter at quantile q for the rth replication and θ is the corresponding 'true' value

(b) Relative Efficiencies (EFF) defined as

$$EFF(\hat{\theta}) = \frac{S^2_{model}(\hat{\theta})}{S^2_{\mathrm{MQ}}(\hat{\theta})}$$

where $S^2(\hat{\theta}) = R^{-1} \sum_{r=1}^{R} (\hat{\theta}^{(r)} - \bar{\theta})^2$ and $\bar{\theta} = R^{-1} \sum_{r=1}^{R} \hat{\theta}^{(r)}$.

Simulation Results

Table: Values of bias (ARB), efficiency (EFF), and average of point estimates over 500 simulations of fixed effects. Methods: MQRE, MQ, LRE at $q =$ (0.5,0.75,0.9)

$q = 0.5$

	$\hat{\beta}_0$ ARB	$\hat{\beta}_0$ EFF	$\bar{\beta}_0$	$\hat{\beta}_1$ ARB	$\hat{\beta}_1$ EFF	$\bar{\beta}_1$
	Scenario 1 - $[N, N]$					
MQRE	-0.000	0.807	100.000	-0.013	0.655	2.000
MQ	0.006	1.000	100.006	-0.034	1.000	1.999
LRE	-0.001	0.774	99.999	-0.016	0.627	2.000
	Scenario 2 - $[T, T]$					
MQRE	0.002	0.827	100.002	-0.018	0.602	2.000
MQ	0.004	1.000	100.004	-0.017	1.000	2.000
LRE	0.001	1.203	100.001	-0.008	0.824	2.000
	Scenario 3 - $[N, Lap]$					
MQRE	0.011	0.857	100.011	-0.050	0.718	1.999
MQ	0.004	1.000	100.004	-0.024	1.000	2.000
LRE	0.009	0.895	100.009	-0.035	0.784	1.999
	Scenario 4 - $[\gamma, \varepsilon]$					
MQRE	-0.004	0.810	99.996	0.025	0.685	2.000
MQ	-0.008	1.000	99.992	0.030	1.000	2.001
LRE	-0.005	1.278	99.995	0.042	1.659	2.001

$q = 0.75$

	$\hat{\beta}_0$ ARB	$\hat{\beta}_0$ EFF	$\bar{\beta}_0$	$\hat{\beta}_1$ ARB	$\hat{\beta}_1$ EFF	$\bar{\beta}_1$
	Scenario 1 - $[N, N]$					
MQRE	-0.189	0.824	101.316	-0.012	0.696	2.000
MQ	-0.185	1.000	101.320	-0.028	1.000	1.999
LRE	—	—	—	—	—	—
	Scenario 2 - $[T, T]$					
MQRE	0.144	0.831	100.910	-0.011	0.607	2.000
MQ	0.144	1.000	100.910	-0.006	1.000	2.000
LRE	—	—	—	—	—	—
	Scenario 3 - $[N, Lap]$					
MQRE	0.165	0.915	101.262	-0.051	0.817	1.999
MQ	0.157	1.000	101.254	-0.024	1.000	2.000
LRE	—	—	—	—	—	—
	Scenario 4 - $[\gamma, \varepsilon]$					
MQRE	0.126	0.825	101.636	-0.011	0.752	2.000
MQ	0.121	1.000	101.631	-0.006	1.000	2.000
LRE	—	—	—	—	—	—

$q = 0.9$

	$\hat{\beta}_0$ ARB	$\hat{\beta}_0$ EFF	$\bar{\beta}_0$	$\hat{\beta}_1$ ARB	$\hat{\beta}_1$ EFF	$\bar{\beta}_1$
	Scenario 1 - $[N, N]$					
MQRE	-0.349	0.849	102.506	-0.017	0.762	2.000
MQ	-0.347	1.000	102.509	-0.030	1.000	1.999
LRE	—	—	—	—	—	—
	Scenario 2 - $[T, T]$					
MQRE	0.230	0.838	101.871	-0.005	0.584	2.000
MQ	0.227	1.000	101.869	0.005	1.000	2.000
LRE	—	—	—	—	—	—
	Scenario 3 - $[N, Lap]$					
MQRE	-0.073	0.960	102.468	-0.039	0.898	1.999
MQ	-0.081	1.000	102.460	-0.018	1.000	2.000
LRE	—	—	—	—	—	—
MQRE	0.613	0.905	103.496	0.002	0.925	2.000
MQ	0.607	1.000	103.490	0.003	1.000	2.000
LRE	—	—	—	—	—	—

Simulation Results

Table: Empirical standard errors and estimated standard errors of $\hat{\beta}_q$ for $q = (0.5, 0.75, 0.9)$ using MQRE with tuning constant $c = 1.345$. The results are based on $R = 500$ Monte Carlo replications

	MQRE			
	Empirical s.e.	Estimated s.e.	Empirical s.e.	Estimated s.e.
	$\hat{\beta}_0$		$\hat{\beta}_1$	
	$q = 0.5$			
Scenario 1 - $[N, N]$	0.2219	0.2189	0.0118	0.0116
Scenario 2 - $[T, T]$	0.1616	0.1567	0.0072	0.0072
Scenario 3 - $[N, Lap]$	0.2060	0.2127	0.0108	0.0105
Scenario 4 - $[\gamma, \varepsilon]$	0.2635	0.2611	0.0143	0.0143
	$q = 0.75$			
Scenario 1 - $[N, N]$	0.2340	0.2288	0.0128	0.0126
Scenario 2 - $[T, T]$	0.1850	0.1806	0.0089	0.0084
Scenario 3 - $[N, Lap]$	0.2212	0.2263	0.0127	0.0121
Scenario 4 - $[\gamma, \varepsilon]$	0.2922	0.2962	0.0169	0.0172
	$q = 0.9$			
Scenario 1 - $[N, N]$	0.2624	0.2559	0.0153	0.0152
Scenario 2 - $[T, T]$	0.2687	0.2649	0.0133	0.0129
Scenario 3 - $[N, Lap]$	0.2725	0.2715	0.0180	0.0171
Scenario 4 - $[\gamma, \varepsilon]$	0.4814	0.4693	0.0326	0.0325

Application

- Data from the Millennium Cohort Study in the UK
- Repeated measures data on same children over 3 waves
- Measure of interest is defined by Strengths and Difficulties Questionnaire (SDQ)
- SDQ score: Sum of the responses on a series of SDQ items that describe a particular behaviour
- SDQ domains of interest: Internalising SDQ (emotional symptoms and peer problems), externalising SDQ (conduct problems and hyperactivity)

Application (Cont'd)

- MCS data is clearly hierarchical
- SDQ distribution is skewed. Model diagnostics from fitting a standard growth curves model indicate lack of normality and a heavy tailed distribution
- Motivating the use of a quantile-type regression
  - Given the asymmetry of the SDQ distribution, use a model for the median rather the mean
  - Examine the effect of different risk factors on the SDQ score not only for kids with average behavioural problems but also for kids with major behavioural difficulties

Diagnostics

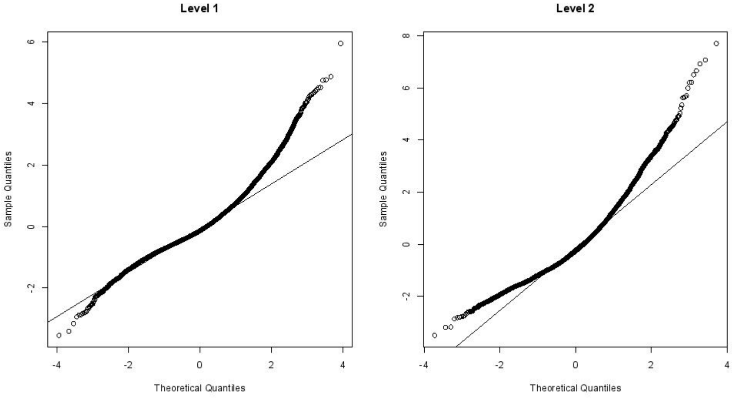


Figure: Normal probability plots of level 1 (left) and level 2 residuals (right) derived by fitting a two level linear mixed model for SDQ internalising problems score.

Diagnostics

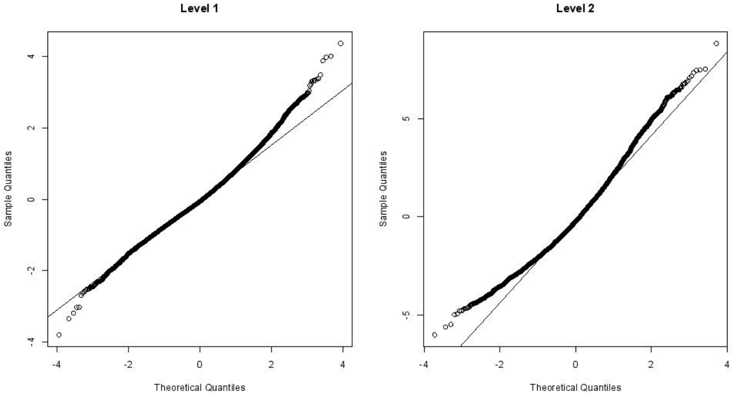


Figure: Normal probability plots of level 1 (left) and level 2 residuals (right) derived by fitting a two level linear mixed model for SDQ externalising problems score.

Results Externalising SDQ - M-quantiles

Table: Results - MQRE random intercepts model for externalising scores. Point estimates, standard errors in parentheses.

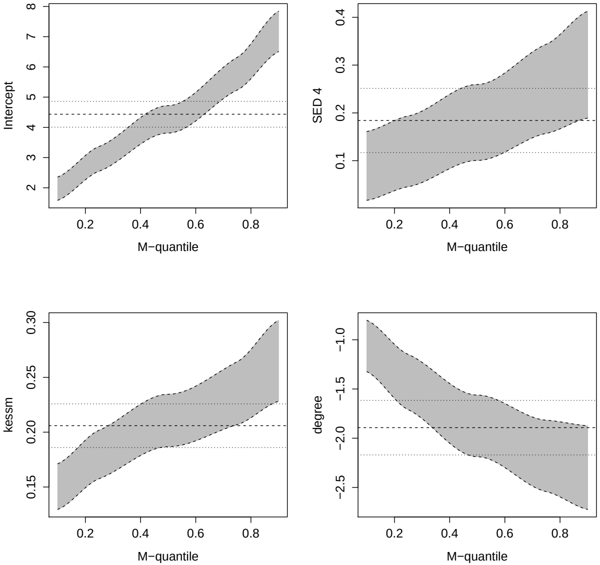
	0.1		0.25		0.5		0.75		0.9	
Intercept	1.972	(0.197)	2.959	(0.208)	4.265	(0.231)	5.774	(0.276)	7.174	(0.341)
age year scal	-0.349	(0.013)	-0.401	(0.013)	-0.456	(0.015)	-0.476	(0.018)	-0.455	(0.023)
age2 year scal	0.136	(0.009)	0.171	(0.009)	0.219	(0.009)	0.258	(0.011)	0.274	(0.016)
ALE 11	0.074	(0.025)	0.098	(0.025)	0.131	(0.027)	0.172	(0.033)	0.208	(0.043)
SED 4	0.089	(0.037)	0.120	(0.038)	0.180	(0.041)	0.250	(0.048)	0.301	(0.057)
kessm	0.150	(0.011)	0.180	(0.011)	0.211	(0.012)	0.236	(0.015)	0.265	(0.019)
degree	-1.063	(0.133)	-1.430	(0.143)	-1.875	(0.160)	-2.180	(0.185)	-2.298	(0.217)
GCSE	-0.421	(0.130)	-0.632	(0.140)	-0.917	(0.154)	-1.078	(0.174)	-1.110	(0.198)
white	0.024	(0.113)	0.061	(0.119)	0.126	(0.135)	0.185	(0.161)	0.179	(0.201)
male	0.658	(0.065)	0.804	(0.071)	0.968	(0.081)	1.094	(0.099)	1.191	(0.121)
imdscore	-0.022	(0.014)	-0.026	(0.015)	-0.035	(0.017)	-0.050	(0.021)	-0.049	(0.026)
Eng eth stratum	0.110	(0.140)	0.212	(0.149)	0.300	(0.168)	0.242	(0.197)	0.139	(0.243)
Eng dis stratum	0.160	(0.083)	0.267	(0.092)	0.401	(0.105)	0.486	(0.131)	0.579	(0.160)
$\sigma^2_{u_q}$	0.708	—	2.564	—	5.718	—	4.754	—	2.127	—
$\sigma^2_{\epsilon_q}$	0.975	—	2.633	—	4.762	—	4.073	—	2.388	—

Results Externalising SDQ - Quantiles

Table: Results - LQMM random intercepts model for externalising scores. Point estimates, standard errors in parentheses.

	0.1		0.25		0.5		LRE - mean		0.75		0.9	
Intercept	1.905	(0.298)	3.174	(0.269)	4.065	(0.274)	4.434	(0.218)	5.033	(0.282)	6.073	(0.328)
age year scal	-0.336	(0.026)	-0.374	(0.019)	-0.433	(0.020)	-0.442	(0.019)	-0.467	(0.019)	-0.478	(0.033)
age2 year scal	0.141	(0.017)	0.162	(0.010)	0.212	(0.011)	0.222	(0.010)	0.255	(0.015)	0.287	(0.018)
ALE 11	0.121	(0.051)	0.128	(0.028)	0.127	(0.043)	0.134	(0.025)	0.187	(0.042)	0.124	(0.046)
SED 4	0.085	(0.060)	0.132	(0.059)	0.252	(0.049)	0.184	(0.034)	0.260	(0.096)	0.294	(0.069)
kessm	0.169	(0.026)	0.208	(0.019)	0.215	(0.016)	0.206	(0.010)	0.252	(0.017)	0.233	(0.023)
degree	-1.212	(0.176)	-1.384	(0.209)	-1.660	(0.180)	-1.893	(0.141)	-1.546	(0.161)	-1.828	(0.191)
GCSE	-0.351	(0.178)	-0.862	(0.166)	-0.829	(0.161)	-0.905	(0.129)	-0.570	(0.167)	-0.743	(0.180)
white	0.119	(0.146)	0.057	(0.138)	0.283	(0.164)	0.142	(0.134)	0.311	(0.135)	0.629	(0.163)
male	0.653	(0.139)	0.756	(0.097)	0.958	(0.077)	0.980	(0.082)	1.189	(0.119)	1.122	(0.124)
imdscore	-0.006	(0.030)	-0.029	(0.026)	-0.037	(0.018)	-0.036	(0.017)	-0.048	(0.031)	-0.035	(0.043)
Eng eth stratum	-0.069	(0.159)	-0.047	(0.209)	0.296	(0.168)	0.271	(0.167)	0.543	(0.166)	0.625	(0.164)
Eng dis stratum	-0.017	(0.111)	0.282	(0.106)	0.282	(0.103)	0.422	(0.106)	0.827	(0.114)	0.903	(0.111)
$\sigma^2_{u_q}$	3.369	—	4.392	—	5.199	—	6.164	—	6.163	—	6.717	—

Results Externalising SDQ - M-Quantiles



Key Findings Externalising SDQ

- Increasing values for the risk factors associated with increased SDQ scores
- Effect of risk factors more pronounced at the upper tail compared to the lower tail of the SDQ distribution
- Disparity in externalising SDQ scores of children with mothers that have higher educational qualifications, compared to children with mothers that have no educational qualifications, is smaller at the lower part of the distribution compared to the upper part of the distribution
- May suggest that the protective role of higher maternal education is more pronounced for children with more externalising problems
- Maternal depression appears to have a more pronounced effect at the top end, compared to the lower end, of the distribution

Results Internalising SDQ - M-quantiles

Table: Results - MQRE random intercepts model for internalising scores. Point estimates, standard errors in parentheses.

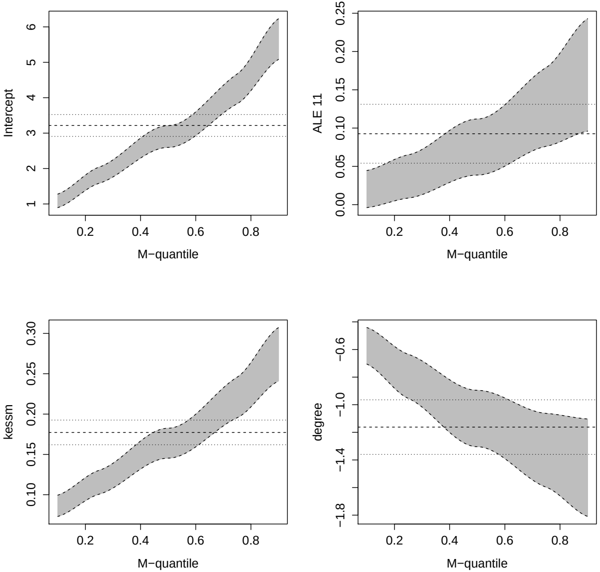
	0.1		0.25		0.5		0.75		0.9	
Intercept	1.088	(0.097)	1.813	(0.116)	2.904	(0.157)	4.249	(0.214)	5.656	(0.293)
age year scal	-0.042	(0.007)	-0.049	(0.008)	-0.043	(0.010)	0.005	(0.014)	0.089	(0.022)
age2 year scal	0.034	(0.005)	0.050	(0.005)	0.075	(0.007)	0.099	(0.009)	0.113	(0.015)
ALE 11	0.020	(0.012)	0.037	(0.014)	0.075	(0.019)	0.128	(0.026)	0.169	(0.037)
SED 4	0.026	(0.018)	0.035	(0.021)	0.061	(0.027)	0.106	(0.037)	0.093	(0.051)
kessm	0.086	(0.007)	0.116	(0.008)	0.164	(0.009)	0.221	(0.013)	0.274	(0.017)
degree	-0.573	(0.068)	-0.794	(0.081)	-1.100	(0.104)	-1.330	(0.136)	-1.456	(0.180)
GCSE	-0.372	(0.065)	-0.531	(0.078)	-0.736	(0.101)	-0.834	(0.131)	-0.813	(0.169)
white	-0.232	(0.058)	-0.327	(0.070)	-0.484	(0.096)	-0.664	(0.132)	-0.805	(0.176)
male	0.062	(0.031)	0.085	(0.038)	0.136	(0.051)	0.190	(0.071)	0.271	(0.100)
imdscore	-0.019	(0.007)	-0.027	(0.008)	-0.047	(0.011)	-0.076	(0.015)	-0.106	(0.023)
Eng eth stratum	0.134	(0.072)	0.204	(0.087)	0.303	(0.115)	0.271	(0.160)	0.159	(0.216)
Eng dis stratum	0.096	(0.039)	0.124	(0.048)	0.149	(0.065)	0.121	(0.094)	0.111	(0.134)
$\sigma^2_{u_q}$	0.147	—	0.631	—	1.958	—	2.108	—	1.074	—
$\sigma^2_{\epsilon_q}$	0.285	—	0.989	—	2.480	—	2.723	—	1.895	—

Results Internalising SDQ - Quantiles

Table: Results - LQMM random intercepts model for internalising scores. Point estimates, standard errors in parentheses.

	0.1		0.25		0.5		LRE - mean		0.75		0.9	
Intercept	1.746	(0.130)	1.680	(0.261)	2.934	(0.161)	3.215	(0.157)	3.870	(0.220)	4.355	(0.184)
age year scal	-0.000	(0.000)	-0.000	(0.009)	-0.037	(0.012)	-0.008	(0.010)	-0.041	(0.016)	-0.063	(0.023)
age2 year scal	0.000	(0.000)	0.000	(0.008)	0.052	(0.011)	0.083	(0.008)	0.096	(0.010)	0.125	(0.015)
ALE 11	0.000	(0.000)	0.000	(0.010)	0.048	(0.021)	0.093	(0.020)	0.104	(0.035)	0.138	(0.034)
SED 4	-0.000	(0.000)	0.000	(0.016)	0.099	(0.039)	0.061	(0.026)	0.071	(0.052)	0.105	(0.051)
kessm	0.000	(0.000)	0.000	(0.034)	0.157	(0.012)	0.177	(0.008)	0.211	(0.016)	0.234	(0.021)
degree	-1.012	(0.061)	-0.687	(0.114)	-0.897	(0.134)	-1.162	(0.101)	-1.142	(0.142)	-0.978	(0.121)
GCSE	-1.012	(0.061)	-0.687	(0.127)	-0.702	(0.116)	-0.718	(0.092)	-0.702	(0.162)	-0.348	(0.124)
white	-0.735	(0.134)	0.008	(0.204)	-0.452	(0.108)	-0.559	(0.095)	-0.499	(0.142)	-0.220	(0.124)
male	0.000	(0.000)	0.000	(0.009)	0.101	(0.059)	0.173	(0.058)	0.153	(0.079)	0.307	(0.099)
imdscore	0.000	(0.000)	0.000	(0.007)	-0.030	(0.014)	-0.059	(0.012)	-0.032	(0.019)	-0.095	(0.017)
Eng eth stratum	-0.276	(0.115)	0.008	(0.074)	0.217	(0.110)	0.273	(0.119)	0.592	(0.148)	0.786	(0.115)
Eng dis stratum	0.000	(0.000)	0.000	(0.016)	0.058	(0.065)	0.148	(0.075)	0.163	(0.097)	0.363	(0.119)
$\sigma^2_{u_q}$	0.000	—	0.750	—	2.280	—	2.722	—	3.562	—	4.240	—

Results Internalising SDQ - M-Quantiles



Key Findings Internalising SDQ

- Results consistent with what the theory of child development predicts
- After controlling for family and area characteristics, socio-economic disadvantage is significantly associated with internalising SDQ scores only at $q = 0.5$ and $q = 0.75$
- This is in contrast to the more pronounced effect of socio-economic disadvantage, across quantiles, we found for externalising SDQ scores
- Maternal depression is significantly associated with increased SDQ internalising scores. This effect is also clearly more pronounced (compared to the results for externalising SDQ scores) at the top end of the distribution
- The protective effect of higher maternal education is present also for internalising problems

R Packages

Table: Quantile, M-quantile & Expectile Multilevel Regression

Model	*R* Package
Quantile (ALD)	*lqmm*, (*lqmm*)
M-Quantile	[1]
Expectile	[2]

[1] Tzavidis et al., 2014

[2] Tzavidis et al., 2014